# Modern Physics

Problem Sheet 4: Commutation relationships, uncertainty and conservation laws

November 15, 2016

NB: For the consistent set of units to be used in these exercise please see the document "Physical Units" in the course webpage.

## Exercise 1: Commutation relationships and Uncertainty principles

- Compute the commutator $[\hat{X}, \hat{P}]$. Hint: use the position representation
- What are the implications on the possibility of simultaneously measure these quantities?
- Using the position representation compute $[\hat{V}, \hat{T}]$ where $\hat{V}$ is the potential energy operator and $T$ is the kinetic energy operator. Show that $T$ and $V$ are not individually conserved by the time evolution, but total energy is.

## Exercise 2: Orbital angular momentum

- Let $\hat{L}$ be the orbital angular momentum in position representation. Compute $[\hat{\vec{L}}^2, \hat{L_z}]$. Can the total magnitude of the angular momentum and one component (for example the one along $\hat{k}$ be simultaneously measured with arbitrary accuracy?
- Consider a quantum particle interacting with a central potential $V = V(|\mathbf{r}|)$. Show that the quantum angular momentum is conserved.

## Exercise 3: Pauli matrixes and spin 1/2 operators

Pauli matrixes are defined as:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

- Show that Pauli matrixes are Hermitian
- Define spin 1/2 matrixes: $\hat{S}_i = \frac{1}{2}\hbar\hat{\tau}_i$, with $i = 1, 2, 3$. Show that spin matrixes obey the same commutation relationship of orbital angular momentum. From here one can identify spin with a type of angular momentum.

## Spin 1/2 eigenstates

Spin 1/2 states can be represented by two-dimensional vectors.

- Show that

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

Are the eigenstates of $\hat{S}_z$.

- Verify that they also are eigenstates of $\hat{S}^2$. Is this a surprise? What are the corresponding eigenvalues?
- What is the result of a measurement of the total spin of a spin 1/2 particle ? (Surprise!!!)